

HSSI 2007 Math —HW #2 Solutions

Problems from §1.3:

4. For $f(x) = 1/x$ and $g(x) = 3x + 4$, find $f(g(1))$, $g(f(1))$, $f(g(x))$, $g(f(x))$, and $f(t)g(t)$.

Well, $g(1) = 7$, so $f(g(1)) = f(7) = 1/7$.

Well, $f(1) = 1$, so $g(f(1)) = g(1) = 7$.

Well, $g(x) = 3x + 4$, so $f(g(x)) = f(3x + 4) = 1/(3x + 4)$.

Well, $f(x) = 1/x$, so $g(f(x)) = g(1/x) = (3/x) + 4$.

Finally, $f(t)g(t) = (1/t)(3t + 4)$.

5. Given the graph of m , graph $n(t) = m(t) + 2$.

You should have the same graph slid up by 2 units.

8. Graph the graph of m , graph $w(t) = m(t - 0.5) - 2.5$.

You should have the same graph slide right by 0.5 units and down by 2.5 units.

22. Let $C = f(A)$ be the cost, in dollars, of building a store of area A square feet. In terms of cost and square feet, what are $f(10000)$ and $f^{-1}(20000)$?

The quantity $f(10000)$ is the cost in dollars of building a store of area 10000 square feet.

The quantity $f^{-1}(20000)$ is the number of square feet you can buy with 20000 dollars.

27. Use the figure to estimate $f^{-1}(2)$ and sketch a graph of f^{-1} .

Reading horizontally to the graph from $y = 2$, we find $x = -1.2$ or so, so $f^{-1}(2) \approx -1.2$. The graph of f^{-1} should be the graph of f reflected over the line $y = x$.

28. Find a possible formula for the given graph.

We can start with a parabola $y = x^2$, and multiply it by -1 to get an upside down parabola $y = -x^2$. Now we want to slide it left by 1 unit, so we replace x with $x + 1$, i.e., $y = -(x + 1)^2$. Finally, we want to slide it up by 3 units, so we add 3, i.e., $y = -(x + 1)^2 + 3$.

29. Find a possible formula for the given graph.

We can start with $y = x^3$ which has this basic shape. Now we want to slide it right by 2 units, so we replace x with $x - 2$, i.e., $y = (x - 2)^3$. Finally, we want to slide it down by 1 unit, so we subtract 1, i.e., $y = (x - 2)^3 - 1$.

Problems from §1.4

5. Simplify $\ln(1/e) + \ln(AB)$.

Since $1/e = e^{-1}$, and $\ln(e^c) = c$ always, we have $\ln(1/e) = -1$. Furthermore, there's a rule that says $\ln(AB) = \ln(A) + \ln(B)$. So, the overall simplified form is $-1 + \ln(A) + \ln(B)$.

9. Solve for x if $20 = 50(1.04)^x$.

First we divide both sides by 50 to get:

$$\frac{20}{50} = (1.04)^x,$$

or

$$0.4 = (1.04)^x.$$

Now we take \ln of both sides and recall that $\ln(A^B) = B \ln A$, so that $\ln(1.04)^x = x(\ln 1.04)$. Thus, the equation becomes:

$$\ln(0.4) = x(\ln(1.04)).$$

Dividing both sides by $\ln(1.04)$, we get the final answer:

$$\frac{\ln(0.4)}{\ln(1.04)} = x.$$

Using a calculator, we can get x in decimal form: $x = -23.36$.

15. Solve for x if $10^{x+3} = 5e^{7-x}$.

First we take \ln of both sides and recall that $\ln(A^B) = B \ln A$, so that $\ln 10^{x+3} = (x+3)(\ln 10)$ and use the product rule to rewrite $\ln(5e^{7-x})$ as $\ln 5 + \ln e^{7-x}$, and then rewrite that as $\ln 5 + (7-x)$ since $\ln e^c = c$ for any c . Thus, the equation becomes:

$$(x+3)(\ln 10) = (\ln 5) + (7-x).$$

It will probably be clearer how to solve this if we plug in decimal values for the lns:

$$(x+3)(2.30) = 1.61 + (7-x).$$

We can add the numbers on the right side to get:

$$(x+3)(2.30) = 8.61 - x.$$

Distributing, we find:

$$2.3x + 6.9 = 8.61 - x.$$

Adding x to both sides, this becomes:

$$3.3x + 6.9 = 8.61.$$

Subtracting 6.9 from both sides, this becomes:

$$3.3x = 1.71.$$

Finally, dividing both sides by 3.3, we get the final answer:

$$x = (1.71)/(3.3) = 0.52.$$

31. Without a calculator or computer, match the functions e^x , $\ln x$, x^2 and $x^{1/2}$ to their graphs in the figure.

Graph A is e^x : it's an increasing and concave up function that passes through $(0, 1)$.

Graph B is x^2 : it's an increasing and concave up function that passes through $(0, 0)$.

Graph C is $x^{1/2}$: it's an increasing and concave down function that passes through $(0, 0)$.

Graph D is $\ln x$: it's an increasing and concave down function that passes through $(1, 0)$.

34. If the size of a bacteria colony doubles in 5 hours, how long will it take for the number to triple?

Bacteria grow according to exponential growth, so the number of bacteria B at time t (in hours) is given by $B(t) = B_0e^{kt}$ for some constant k (and B_0 is the initial number of bacteria).

What does it mean that the colony doubles in 5 hours? It means that when $t = 5$, we have $B = 2B_0$ (twice what we started with). Thus:

$$2B_0 = B_0e^{k(5)}.$$

Dividing each side by B_0 , this becomes:

$$2 = e^{5k}.$$

Taking \ln of each side, this becomes

$$\ln 2 = 5k,$$

or $k = (\ln 2)/5 = 0.138629$.

Now we want to know the time for the colony to triple. This means we want to know for what t we will have $B = 3B_0$. So,

$$3B_0 = B_0e^{0.138629t}.$$

Dividing each side by B_0 , this becomes:

$$3 = e^{0.138629t}.$$

Taking \ln of each side, this becomes

$$\ln 3 = 0.138629t,$$

or $t = (\ln 3)/(0.138629) = 7.92$ hours.

42. In 1980, there were about 170 million vehicles and about 227 million people in the US. The number of vehicles has been growing at 4 % per year, while the population has been growing at 1 % per year. When was there, on average, one vehicle per person?

“Growth at 4 % per year” means that the number of vehicles is $V(t) = 170000000(1.04)^t$ where t is years after 1980. Similarly, the population is $P(t) = 227000000(1.01)^t$.

We want to know the value of t when these are equal:

$$170000000(1.04)^t = 227000000(1.01)^t.$$

Taking \ln of both sides, we find:

$$\ln(170000000) + \ln(1.04)^t = \ln(227000000) + \ln(1.01)^t.$$

Using a basic law of \ln , this can be rewritten as:

$$\ln(170000000) + t(\ln(1.04)) = \ln(227000000) + t(\ln(1.01)).$$

Next we replace all the \ln terms by decimal versions:

$$18.95131 + 0.039221t = 19.24046 + 0.00995t.$$

Next we subtract $0.00995t$ from each side, to get:

$$18.95131 + 0.02927t = 19.24046.$$

Next we subtract 18.95131 from each side, to get:

$$0.02927t = 0.289152.$$

Finally, we divide each side by 0.02927 to get $t = 9.88$ years, i.e., toward the end of 1989.