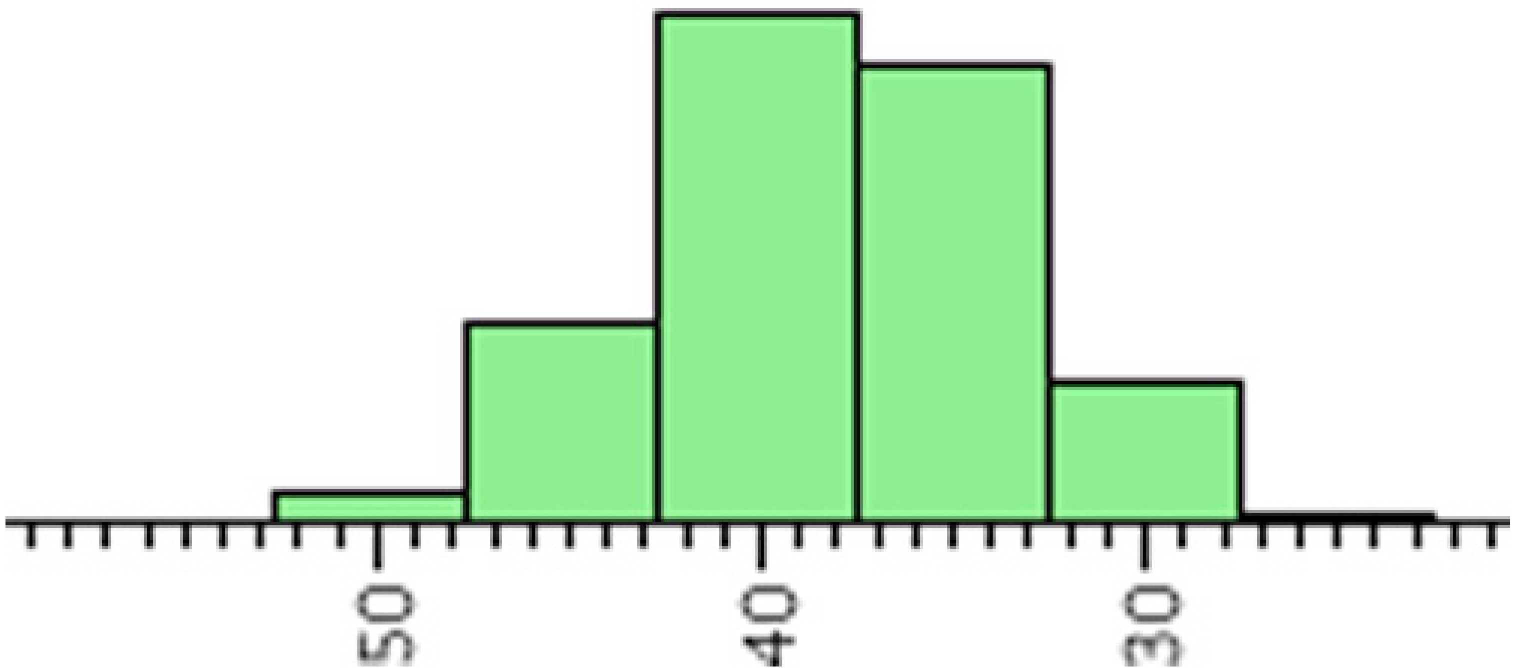
Formulas for the expected value and variance of continuous random variables are deduced immediately from formulas for discrete random variables, because any continuous rv can be approximated by discrete random variables:

Population: Slices, excluding the ends, of Oatnut
Experiment: Use Lynne's kitchen scale to determine the mass, in grams, of a slice
Sample space: {0, 5, 10, 15, 20, 25, 30, 35, 40 ,…}
Random variable: $P(X = x)$ is the probability that a slice has mass between $x - 2.5$ grams and $x + 2.5$ grams

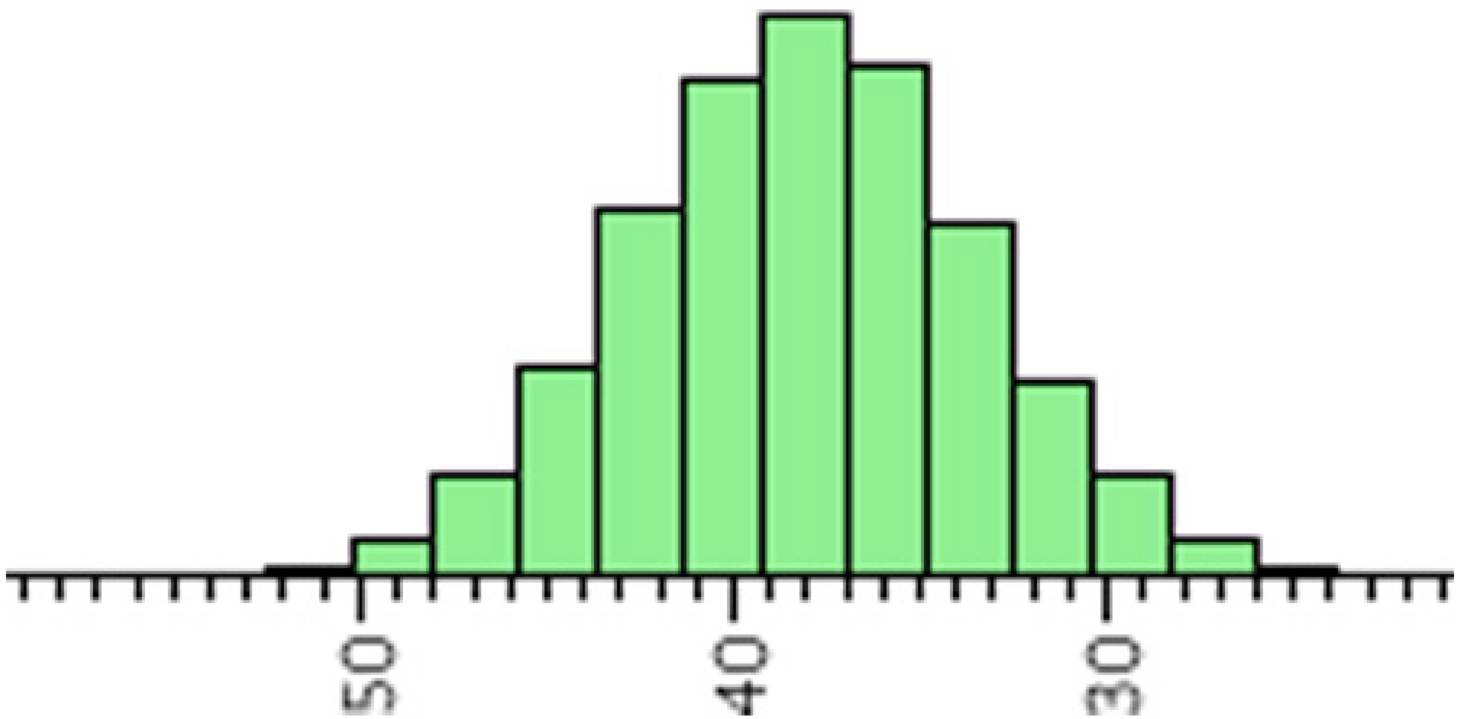
To get a better approximation to the continuous random variable whose distribution is the distribution of masses of slices of Oatnut, use a better scale!

Distribution of Masses as Determined by Lynne’s Kitchen Scale

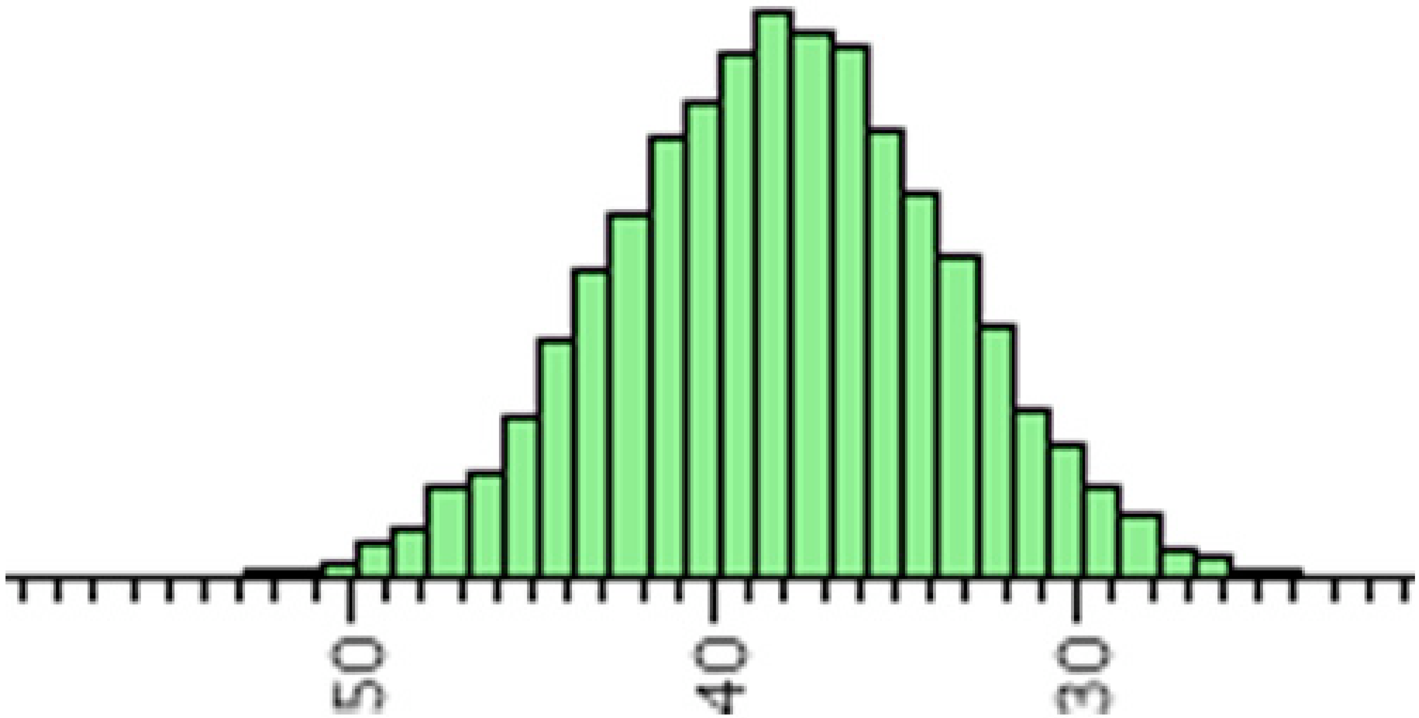


1 22 220 1504 3722 3392 1026 107 6

Distribution of Masses as Determined by a Better Scale



Distribution of Masses as Determined by an Even Better Scale



Think of the normal distribution this approximates as the distribution of masses in the population as measured by a perfect scale.

The True Distribution of Masses

The masses used to generate all three distributions were rounded values from a sample of size n =10,000 from a normal distribution with $\mu = 38.26474$ and $\sigma = 4.73885$.

For this sample,

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = 38.21$$

$$s = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n - 1}} = 4.71$$