

Mathematics 104

Fall 2002

Assignment #2 Due 9pm Wednesday, September 18

Reading: TOTC: 2-9

1. Explain, using the fact that $\sqrt{2}$ is irrational, why $\frac{3}{4}\sqrt{2}$ is irrational. Hint: Use proof by contradiction. Assume that $\frac{3}{4}\sqrt{2}$ is *rational*, and so can be written as p/q for integers p and q , then show that this implies that $\sqrt{2}$ is rational, giving a contradiction.
2. Explain why $2 - \sqrt{2}$ is irrational, using the proof for $2 + \sqrt{2}$ provided in class.
3. Show that the sum of two rational numbers is always rational. Do this by writing $p/q + r/s$ and combining the fractions using the lowest common denominator. Be sure to explain why the result is rational.
4. Show, by example, that the sum of two irrational numbers can be either rational or irrational. Hint: Try $2 + \sqrt{2}$ and $2 - \sqrt{2}$.
5. Show, by example, that the product of two irrational numbers can be rational.
6. Express the repeating decimal $123.456456456\dots$ as the ratio of two integers p/q .

Writing: Write a one- to two-page essay explaining, with examples, the meaning of the words *commensurable*, *rational* and *irrational*. Finish your essay by writing out the proof that $\sqrt{2}$ is irrational. Be sure that you understand all of the steps in the proof, and try to write it in your own words, instead of simply copying from your class notes.