

# Topology II (Math 336)

Spring 2010

## 1 Technicalities

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**Course web page:** <http://www.haverford.edu/math/cshonkwi/teaching/m336s10>

**Texts:** *Algebraic Topology*, by Allen Hatcher. We will be using parts of Chapters 0 and 2, which are available *free* online at: <http://www.math.cornell.edu/~hatcher/AT/ATchapters.html>  
You're also, of course, welcome to purchase the book, which is not too expensive.

In the second part of the course, we will use articles and selections from the following books, which are on reserve in the library:

- Rolfsen, *Knots and Links*
- Hempel, *3-manifolds*
- Stillwell, *Classical Topology and Combinatorial Group Theory*
- Birman, *Braids, Links, and Mapping Class Groups*

**Office Hours:** Tentatively Monday 1:00–2:00, Wednesday 2:00–4:00 and Thursday 1:00–2:00.

## 2 Summary of the Course

Last semester, you examined point-set topology, which provides the language to formulate questions about nearness and continuity, and the fundamental group, which provides a way of studying those questions using algebraic tools. This semester, we will deepen our ability to formulate and solve topological problems in three ways:

1. There are two problems with the fundamental group: first, it is very difficult to compute, and second, it only sees low-dimensional properties of a space (for example, the fundamental group cannot distinguish the two-dimensional sphere from the three-dimensional sphere). To overcome these difficulties, we will develop a new topological invariant called the **homology** of a space. While the definition of homology can seem a bit esoteric, it is much more computable than the fundamental group and sees higher-dimensional phenomena.
2. Last semester you touched on some general methods for constructing complicated topological spaces out of simple ones: simplicial complexes and cell complexes. These general constructions will become more important this semester, though we will shift from simplicial complexes to  **$\Delta$ -complexes** and from cell complexes to **CW-complexes**.
3. We will discuss some of the fundamental constructions and problems in low-dimensional topology. I intend to be somewhat flexible about exactly what topics we will cover, but some things we might get to are:
  - **Knot theory**, particularly the **Alexander polynomial** and its relation to the **genus** of a knot.
  - **Mapping Class Groups**, i.e. groups of self-homeomorphisms of surfaces and their relationship with **braids**.

- **3-manifolds** and how to represent them by **Heegaard splittings** and **surgery**. I hope to be able to end with the following theorem:

Every closed, orientable 3-manifold can be constructed by surgery on a link in  $S^3$ .

## 3 Assignments

### 3.1 Homework

There will be weekly homework assignments, typically due at the beginning of class on Friday. In relation to Josh's classification system, assignments will consist entirely of problems, though I can suggest exercises if you feel you need more practice with the definitions.

The grading rubric is the same as Josh's, namely:

- 4 points:** A clear and completely correct solution of the problem.
- 3 points:** The solution demonstrates a good understanding of the problem, but the solution is not clearly written or some details are incorrect or missing.
- 2 points:** The solution is incorrect or incomplete, but a reasonable strategy for solving the problem is in place.
- 1 point:** The solution is incorrect or incomplete, with no reasonable strategy for solving the problem, but it is clear that some thought went into it.
- 0 points:** No reasonable attempt at a solution is given.

Any problem may be rewritten for the average of the old and new scores. Rewrites are due one week after your homework is returned to you. I will accept one late homework assignment (up to the Monday after the original due date) provided you let me know in advance.

Collaboration on homework is encouraged but not necessary. The process of doing mathematics is a strange mixture of the social and the antisocial: it is often the case that you will understand a problem and its solution better when you discuss it with someone else, but it is also essential that you work through the details on your own so that you don't use another's understanding as a crutch. As such, you should follow these guidelines when collaborating:

- You must work on each problem on your own before discussing it with your collaborators.
- When discussing a problem with your collaborators, you should work on a white board, a blackboard, or a colored sheet of paper.
- You should write up your solution separately from your collaborators and without reference to the white board, blackboard, or colored paper containing your joint work so as to communicate your own understanding of the problem.
- You must indicate with whom you collaborated on your homework. Who you work with may change from problem to problem, so please note your collaborators separately for each problem.

The distinction between use and abuse of online resources in a class like this is hard to make *a priori*. Looking up definitions is fine, reading another person's solution to a problem obviously is not. If you have any questions or doubts, please ask me.

### 3.2 Class Presentations

Some portion of each Friday class—from ten minutes to the entire meeting—will be reserved for student presentations. The material for the presentations will consist of extra examples, proofs of lemmata that I skipped, and short explorations into applications and related topics. Some of the material will be in Hatcher but in need of more careful explanation; some will require some work (though generally it will not be as hard as the problems).

The purpose of these presentations is twofold: first, you'll thank me for the extra speaking practice when you have to give your senior thesis presentation. Second, I want to blur the line between the mathematics that I present and the problems you do to understand it: *you* are now partly responsible for the theoretical content.

The topics for the presentations will be assigned with the problem set on Monday, and you will volunteer for the presentations by Wednesday.

## 4 Exams:

There will be a midterm exam and a final. The midterm will take the place of a homework assignment (handed out Monday, due Friday), and will be untimed and open-book. The final will also be untimed and open-book and will be implicitly cumulative. Collaboration is not allowed on either of the exams.

## 5 Grading

Your final grade in the course will be determined by:

**Homework:** 50%

**Presentations:** 10%

**Midterm:** 20%

**Final:** 20%

## 6 Accommodations

If you think you may need accommodations in this course due to the impact of a disability please meet with me privately as soon as possible. You should also contact Rick Webb, Coordinator, Office of Disabilities Services ([rwebb@haverford.edu](mailto:rwebb@haverford.edu), 610-896-1290) to confirm your eligibility for appropriate accommodations. Doing so early in the semester will help prevent unnecessary inconvenience.