

Math 115 Exam #3 Practice Problems

1. Solve the initial-value problem $\frac{dx}{dt} + 2tx = x$, $x(0) = 5$. Use your solution to compute $x(3)$.
2. Solve the differential equation $7yy' = 5x$.
3. Solve the initial-value problem $y' + y = 2$, $y(0) = 1$.
4. Solve the initial-value problem $\frac{dy}{dx} = 1 - y + x^2 - yx^2$, $y(0) = 0$.
5. Solve the differential equation $x\frac{dy}{dx} = y^2$.
6. Solve the differential equation $(x^2 + 1)\frac{dy}{dx} = y$.
7. Solve the initial-value problem $\frac{dy}{dx} = xy$, $y(1) = 3$.
8. In a second-order chemical reaction, the reactant A is used up in such a way that the amount of it present decreases at a rate proportional to the square of the amount present. Suppose this reaction begins with 50 grams of A present, and after 10 seconds there are only 25 grams left. How long after the beginning of the reaction will there be only 10 grams left? Will all of the A disappear in a finite time, or will there always be a little bit present?
9. Market research has shown the price p and weekly sales $S(p)$ of a particular product are related by the following differential equation:

$$\frac{dS}{dp} = -\frac{1}{2} \left(\frac{S}{p+3} \right).$$

If sales amount to 100 units when the price is \$1 (i.e. $S(1) = 100$), what will the weekly sales be if the price is raised to \$6?

10. Solve the initial-value problem $\frac{dy}{dx} = \frac{e^{2x}}{6y^5}$, $y(0) = 1$.
11. If $y(x)$ satisfies the differential equation $\frac{dy}{dx} = e^{2x-y}$ and $y(0) = 1$, then what is $y(1/2)$?
12. Solve the differential equation $x^2y' - y = 2x^3e^{-1/x}$.
13. Solve the initial-value problem $xy' - y = x \ln x$, $y(1) = 2$.
14. Solve the initial-value problem $(x^2 + 1)\frac{dy}{dx} + 3x(y - 1) = 0$, $y(0) = 2$.
15. Solve the differential equation $t \ln t \frac{dr}{dt} + r = te^t$.
16. In the following predator-prey system, determine which of the variables, x or y , represents the prey population and which represents the predator population. Do the predators feed only on the prey or do they have additional food sources? Explain.

$$\begin{aligned}\frac{dx}{dt} &= -0.05x + 0.0001xy \\ \frac{dy}{dt} &= 0.1y - 0.005xy\end{aligned}$$