

### Math 115 Exam #1 Practice Problems

For each of the following, say whether it converges or diverges and explain why.

1.  $\sum_{n=1}^{\infty} \frac{n^3}{n^5+3}$

2.  $\sum_{n=1}^{\infty} \frac{3^n}{4^n+4}$

3.  $\sum_{n=1}^{\infty} \frac{n}{2^n}$

4. For what values of  $p$  does the series  $\sum_{n=1}^{\infty} \frac{n^p}{2+n^3}$  converge?

5. We would like to estimate the sum of the series  $\sum_{n=1}^{\infty} \frac{1}{n^4+3}$  by using the sum of the first ten terms. Of course, the exact error is the sum of all the terms from the 11th on, i.e.,  $\sum_{n=11}^{\infty} \frac{1}{n^4+3}$ . Show that this error is less than  $1/3000$  by comparing this with the sum of  $1/n^4$  and then by estimating this latter sum using an appropriate integral.

6. Does the series

$$\sum_{n=1}^{\infty} \frac{n!(n+1)!}{(3n)!}$$

converge or diverge?

7. Does the series

$$\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{1}{n}\right)$$

converge absolutely, converge conditionally, or diverge?

8. Determine the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{n^3 x^{3n}}{n^4 + 1}$$

9. Consider the sequence defined by  $a_n = \frac{(-1)^{n+n}}{(-1)^{n-n}}$ . Does this sequence converge and, if it does, to what limit?

10. Find the value of the series

$$\sum_{n=1}^{\infty} \frac{1+2^n}{3^{n-1}}$$

11. Does the series

$$\sum_{n=1}^{\infty} \frac{n+5}{n\sqrt{n+3}}$$

converge or diverge?

12. Does the series

$$\sum_{n=1}^{\infty} \frac{3 + \cos n}{e^n}$$

converge or diverge?

13. Does the series

$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{\sqrt{n^2 + 1}}$$

converge absolutely, converge conditionally, or diverge?

14. Does the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{n!}{\pi^n}$$

converge absolutely, converge conditionally, or diverge?