

# A physical analysis of mechanical computability

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## Mathematical or Physical?

Are computing machines abstract models or concrete devices?

Both!

Try to reconcile:

1. The mathematical notion of an algorithm.
2. The physical notion of a computation.

How do the logical limits differ in each case?

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## Answers

- Most talks: questions ... answers
- This talk: answers ... questions
- Popular misconception of Turing machine: that it is a physical model of computation
- Logically universal – physically realizable?

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## ‘Abstract’ model

Alan Turing (1936)

- “automatic machine” – motion is “*completely* determined by the [internal] configuration”
- *no external influences* of any form: neither data (e.g. oracular assistance) nor instructions (e.g. random bits)

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## Conceptual Outline

- An analysis of *mechanical* computability
  - Mental perspective (memory & thoughts)
  - Physical perspective (mass & energy)
- With emphasis on its *self-contained* nature
  - automatic (closed w.r.t. flow of information)
  - autonomous (closed w.r.t. material resources)
- Toward a notion of *feasible* algorithms
  - computability and complexity intertwined
  - Are there ‘natural’ asymptotic limits?

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## Mental calculator

- Turing analyzed (an idealized form of) routine *human* calculation performed in a mentally automatic fashion (called ‘computers’ < 1950)
- Captured the *psychological* aspects of ‘mechanical’ computation in which the memory and thought process were self-contained
- Mechanical – ‘in a routine manner’

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## Turing's assumptions

- fixed number of “immediately recognisable” symbols written on a tape divided into “squares”
- fixed number of states “of mind” (not “arbitrarily close”) operating sequentially in a “simple” fashion – altering at most one symbol based upon “observed” squares (locality)
- *No limit* on the number of “tape squares” (cells) or “simple operations” (steps) allowed

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## Computational complexity

Goal: **quantify** the “difficulty” of computing as a function of the size of the input  $n$ .

$S(n)$  *memory* = amount of *room* on the tape

$T(n)$  *thought* = amount of *delay* in the control

ignore constants of proportionality by only looking at *rates of growth* [Hartmanis, et al.]

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## Logical feasibility

Identified those aspects *invariant* of size  
Allowed for computations of *arbitrary* size

Uniform: intensive quantities must be bounded

- finite symbol alphabet; finite state control

Scalable: extensive quantities are not bounded

- potentially infinite memory (occupied cells)
- potentially infinite thoughts (performed steps)

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## Space & Time as resources

There is an implicit assumption that:

- Information (remembering) requires *space*
- Computation (thinking) necessitates *time*

Measure the growth of the (relevant) extensible properties in the mathematical model modulo its (irrelevant) intensive variations.

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## *Mechanical* computability

- Premise: it is a *mathematical* notion
- ‘Abstract’ model – Mental calculator
- Turing’s assumptions – *uniform & scalable*
- Logical feasibility

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## Rulers and Stopwatches

Number of cells  $S(n)$  counted corresponds to volume of physical space  $s$  measured.

$$S(n) \sim s$$



Number of steps  $T(n)$  counted corresponds to length of physical time  $t$  measured.

$$T(n) \sim t$$



- explicitly relates discrete (*countable*) quantities in the mathematical model to continuous (*measurable*) quantities in the physical world

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## Scales and Meters

Weigh the materials



Symbols not only occupy space, but also use *matter* to represent patterns.

Gauge the effort



State transitions take time, but also consume *energy* to perform their transformations.

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## Automaton

*(from autoz, self, and maw, to seize): a self-moving machine, or one in which the principle of motion is contained within the mechanism itself. According to this description, clocks, watches and all machines of a similar kind are automata, but the word is generally applied to contrivances which simulate for a time the motions of animal life. —Encyclopædia Britannica, 11th edition (1911)*

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## Energy & Mass as resources

Grow with length and size – *extensive*

Corresponding *intensive* quantities:

- Power – energy per unit time
- Density – mass per unit space
- The technology (built & operated) is fixed

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## Physical limits



“Classical Thermodynamics is the only physical theory of universal content which .... within the framework of its basic notions, will never be toppled.” – *Albert Einstein*

First: Conservation of energy for a closed system (no exchange of matter with surroundings)

Second: Entropy cannot decrease in an isolated system (no exchange of matter or energy)

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## Physical feasibility

Uniformity – *density* and *power* are bounded over the size and length of the computation

Scalability – the total *mass* and *energy* available are unbounded in space and time

Mass-energy versus space-time complexity:

- What's the difference?

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## Conservation

Mass and energy are *conserved* quantities!  
So for a machine to be autonomous:

- Self-sufficient: the input (padded as necessary) must include these resources
- distribution must be uniformly bounded (hence linearly related, even allowing for conversion between matter and energy)

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## Physical calculator

- Propose analyzing (an idealized form of) routine *machine* calculation performed in a physically autonomous fashion
- Try to capture the *physiological*\* aspects of 'mechanical' computation in which the material resources are self-contained

\* Relating to the material universe or natural science; physical (*Obs.*)

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## Abstract versus Concrete Analysis

Feasible model: Uniform Scalable  
*Intensive* *Extensive*  
**System** *bounded* *unbounded*

<u>Mathematical</u> (discrete)	<b>Tape</b>	# symbols	# cells
	<b>Head</b>	# states	# steps

<u>Physical</u> (continuous)	<b>Memory</b>	<i>Density</i>	<i>Mass</i>
	<b>Processor</b>	<i>Power</i>	<i>Energy</i>

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## *Mechanical* computability

- Premise: it is a *physical* notion
- 'Real' computers require mass and energy
- Conservation of resources – autonomy
- Physical model – self-contained

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## Locality of resources

- Consider an Aristotelian model in which the movement of energy (or mass) entails a small percentage cost
- Availability drops exponentially with radius, but the amount accessible increases only polynomially (in finite-dimensional space)
- Finite-visit property in closed model

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## Locality considerations

- Space-Time: don't permit communication between cells of unbounded distance in one step because information cannot travel faster than the speed of light
- Forbids multi-head processor, random-access memory, or any other non-local model

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## Finite-visit computations

- On a 1-D tape = finite state automata
- On a 2-D tape = full Turing machine
- Turing's claim that there is no essential difference between them requires further investigation *from a physical point of view*

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## Inertial models

- Use Galilean physics to transfer energy over long distances without attenuation
- Time is no longer proportional to # of steps
- Infinite visits – where does all the heat go?

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## Gravitation

- Maximum concentration of mass occurs in a black hole – mass proportional to radius
- Holographic bound – entropy proportional to surface area (square of mass or radius)

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## Newtonian physics

- matter is self-attracting (astronomical size)
- Forbids a uniformly spaced 2-D grid (cellular automata!)
- Non-uniform distribution of mass might be OK, but space is no longer proportional to # cells

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## Questions

- mechanical computability viewed from a mathematical or physical perspective
- a Church's thesis for physical models?
- choose which theory of the "real" world?
- a logically universal physical machine?
- equal to our mathematical *machines*?

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## Thermodynamics

- Temperature is in intensive quantity
- Destroying information produces heat (because of local entropy reduction)
- Finite-visit computations are reversible

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