A physical analysis of mechanical computability

Steven Lindell
Department of Computer Science
Haverford College

Answers

- Most talks: questions … answers
- This talk: answers … questions
- Popular misconception of Turing machine: that it is a *physical model* of computation
- Logically universal – physically realizable?

Conceptual Outline

- An analysis of *mechanical* computability
  - Mental perspective (memory & thoughts)
  - Physical perspective (mass & energy)
- With emphasis on its *self-contained* nature
  - automatic (closed w.r.t. flow of information)
  - autonomous (closed w.r.t. material resources)
- Toward a notion of *feasible* algorithms
  - computability and complexity intertwined
  - Are there ‘natural’ asymptotic limits?

Mathematical or Physical?

Are computing machines abstract models or concrete devices?

Both!

Try to reconcile:

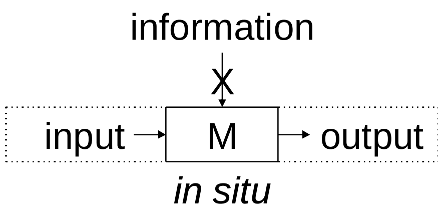
1. The mathematical notion of an algorithm.
2. The physical notion of a computation.

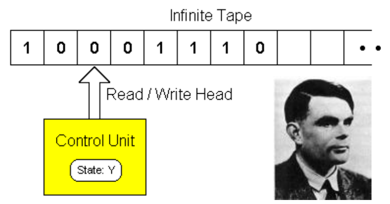
How do the logical limits differ in each case?

‘Abstract’ model

Alan Turing (1936)

- “automatic machine” – motion is “*completely* determined by the [internal] configuration”





- *no external influences* of any form: neither data (e.g. oracular assistance) nor instructions (e.g. random bits)

Mental calculator

- Turing analyzed (an idealized form of) routine *human* calculation performed in a mentally automatic fashion (called ‘computers’ < 1950)
- Captured the *psychological* aspects of ‘mechanical’ computation in which the memory and thought process were self-contained
- Mechanical – ‘in a routine manner’

Turing's assumptions

- fixed number of "immediately recognisable" symbols written on a tape divided into "squares"
- fixed number of states "of mind" (not "arbitrarily close") operating sequentially in a "simple" fashion – altering at most one symbol based upon "observed" squares
- *No limit* on the number of "tape squares" (cells) or "simple operations" (steps) allowed

Logical feasibility

Identified those aspects *invariant* of size
Allowed for computations of *arbitrary* size

<u>Uniform</u>: intensive quantities must be bounded
- finite alphabet of symbols; finite control of states

<u>Scalable</u>: extensive quantities are not bounded
- potentially infinite memory (occupied cells)
- potentially infinite thoughts (performed steps)

Mechanical computability

- Premise: it is a *mathematical* notion
- 'Abstract' model – Mental calculator
- Turing's assumptions
- Logical feasibility

'Random' access memory

Turing: "Intelligent Machinery" (1948)

- Logical Computing Machines (LCM's)
  – claims they are chiefly of theoretical interest because of unlimited time and storage
- Practical Computing Machines (PCM's)
  – bound on size and duration known in advance
  – proposes switched memory organization
  – discusses why their capacity is not extendible
  – concludes they are not 'universal', like LCM's

Computational complexity

<u>Goal</u>: **quantify** the “difficulty” of computing as a function of the size of the input *n*.

S(*n*) *memory* = amount of *room* on the tape
T(*n*) *thought* = amount of *delay* in the control

ignore constants of proportionality by only looking at *rates of growth* [Hartmanis, et al.]

Space & Time as resources

There is an implicit assumption that:

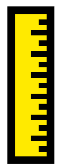
- Information (remembering) requires *space*
- Computation (thinking) necessitates *time*

Measure the growth of the (relevant) extensible properties in the mathematical model modulo its (irrelevant) intensive variations.

Rulers and Stopwatches

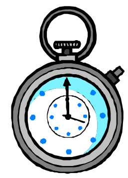
Number of cells *S*(*n*) counted corresponds to volume of physical space *s* measured.

S(*n*) ~ *s*



Number of steps *T*(*n*) counted corresponds to length of physical time *t* measured.

T(*n*) ~ *t*



- explicitly relates discrete (*countable*) quantities in the mathematical model to continuous (*measurable*) quantities in the physical world

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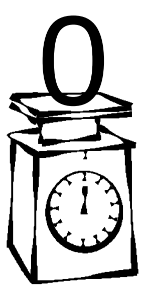
Scales and Meters

Weigh the materials



Symbols not only occupy space, but also use *matter* to represent patterns.

Gauge the effort

$0 \rightarrow 1$

State transitions take time, but also consume *energy* to perform their transformations.

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Energy & Mass as resources

Grow with length and size – *extensive*

Corresponding *intensive* quantities:

- Power – energy per unit time
- Density – mass per unit space

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Physical feasibility

Uniformity – *density* and *power* are bounded over the size and length of the computation

Scalability – the total *mass* and *energy* available are unbounded in space and time

Mass-energy versus space-time complexity:

- What's the difference?

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Automaton

(from autoz, self, and maw, to seize): a self-moving machine, or one in which the principle of motion is contained within the mechanism itself. According to this description, clocks, watches and all machines of a similar kind are automata, but the word is generally applied to contrivances which simulate for a time the motions of animal life. —Encyclopædia Britannica, 11th edition (1911)

Conservation

Mass and energy are *conserved* quantities!

So for a machine to be autonomous:

- Self-sufficient: the input (padded as necessary) must include these resources
- distribution must be uniformly bounded (hence linearly related, even allowing for conversion between matter and energy)

Physical calculator

- Propose analyzing (an idealized form of) routine *machine* calculation performed in a physically autonomous fashion
- Try to capture the *physiological** aspects of ‘mechanical’ computation in which the material resources are self-contained

* Relating to the material universe or natural science; physical (*Obs.*)

Mechanical computability

- Premise: it is a *physical* notion
- ‘Real’ computers require mass and energy
- Conservation of resources – autonomy
- Physical model – self-contained

Abstract versus Concrete Analysis

Feasible model:	System	*Uniform* / *Intensive* / *bounded*	*Scalable* / *Extensive* / *unbounded*
Mathematical (discrete)	**Tape**	*# symbols*	*# cells*
	Head	*# states*	*# steps*
Physical (continuous)	**Memory**	*Density*	*Mass*
	Processor	*Power*	*Energy*

Locality of resources

- Consider an Aristotelian model in which the movement of energy (or mass) entails a small percentage cost
- Availability drops exponentially with radius, but the amount accessible increases only polynomially (in finite-dimensional space)
- Finite-visit property in closed model

Finite-visit computations

- On a 1-D tape = finite state automata
- On a 2-D tape = full Turing machine
- Turing's claim that there is no essential difference between them requires further investigation *from a physical point of view*

Inertial models

- Use Galilean physics to transfer energy over long distances without attenuation
- Time is no longer proportional to # of steps
- Infinite visits – where does all the heat go?

Newtonian physics

- matter is self-attracting (astronomical size)
- Forbids a uniformly spaced 2-D grid (cellular automata!)
- Non-uniform distribution of mass might be OK, but space is no longer proportional to # cells

Thermodynamics

- Temperature is in intensive quantity
- Destroying information produces heat (because of local entropy reduction)
- Finite-visit computations are reversible

Gravitation

- Maximum concentration of mass occurs in a black hole – mass proportional to radius
- Holographic bound – entropy proportional to surface area (square of mass or radius)

Questions

- mechanical computability viewed from a mathematical or physical perspective
- a Church’s thesis for physical models?
- choose which theory of the “real” world?
- a logically universal physical machine?
- equal to our mathematical *machines*?