

**Exam Guidelines:**

1. **IMPORTANT:** Shortly before beginning the exam, you must check your e-mail, and also send an e-mail to [wsmith@haverford.edu](mailto:wsmith@haverford.edu) to notify me that you're about to start. This way, if there are last minute corrections, it will be easy to distribute them to the rest of the class. If you find something on the exam which is erroneous or ambiguous, call me immediately (Office: 610-896-1332, Home 610-896-1565), even if it's very late at night! Although I try extremely hard to make everything completely clear, it is sometimes difficult to anticipate the way that others might interpret the problems. If you call me, we can straighten it out, and using the e-mail mechanism above, we can **straighten it out for the other students, also.**
2. This is a 105 minute, take-home exam. It must be completed in one continuous sitting (i.e., the clock doesn't stop if you take a break).
3. You may prepare one sheet with up to 30 equations to refer to during the test; this may not contain any words or figures. Other than this one sheet, no books, notes, etc. of any kind are permitted.
4. **You should have a calculator to take this exam.** If your calculator has graphing, programmable, or symbolic algebra/calculus features, you may not use these during the exam.
5. This exam contains 4 problems. When you're ready to begin, please check that all are present.
6. For each problem, circle your final answer.
7. The exam is to be slid under my office door (at HC or BMC) by 4 pm Sunday. Please do not turn in this exam itself or your equation sheet; only turn in your examination book. You should carefully keep this exam for future reference when your graded exam is returned.
8. Please note your starting and ending times on the front cover of the examination book.
9. Use three significant digits on all problems, unless otherwise indicated.

**Some Words of Advice:**

I look forward to giving you partial credit for your work. To receive credit, present your work in a clear, readable format. If you find yourself stuck on a problem, don't panic. Instead, carefully explain what you do know about the problem, what you think is going on, and how you might proceed if you could somehow get yourself "unstuck" from the part of the problem that is giving you trouble. Remember, make sure you clearly explain what you write down, since I cannot give partial credit for things I find ambiguous, or for simply copying down equations from your summary sheet. **SHOW ALL YOUR WORK! SHOW ALL YOUR WORK IN THE EXAMINATION BOOKLET!** Be sure to label your work with the problem number.

**IMPORTANT: YOU WILL FIND SOME MULTIPLE-PART PROBLEMS ON THE EXAM. EVEN IF YOU CAN'T GET THE FIRST PART OF SUCH A PROBLEM, IT IS ALMOST ALWAYS POSSIBLE TO GO ON AND DO THE OTHER PARTS!!!!!!!!!!**

There will be some time pressure on this exam. Use the time available wisely.

**There are 100 points total available on the exam.**

**After You've Completed the Exam:**

1. Indicate your ending time on the front cover of your examination booklet.
2. Please sign the honor code pledge on the front cover of your examination booklet.
3. Please do not discuss ANY aspect of this exam with your classmates until I tell you it's okay (this includes, for example, even such things as whether the exam was easy or difficult, long or short, etc.).

**1. (40 points total)** Particle 1 is a spin- $\frac{1}{2}$  particle, and particle 2 is a spin-1 particle. We ignore all degrees of freedom other than spin.

**a. (10 points)** Using the one-particle eigenstates of  $\hat{S}_z$  and  $\hat{S}^2$ , along with the direct product, form a basis for this two-particle system. How many entries are there in the column matrix used to represent a two-particle state? For an arbitrary two-particle state  $|\psi\rangle$ , indicate the inner product corresponding to each entry in this column matrix.

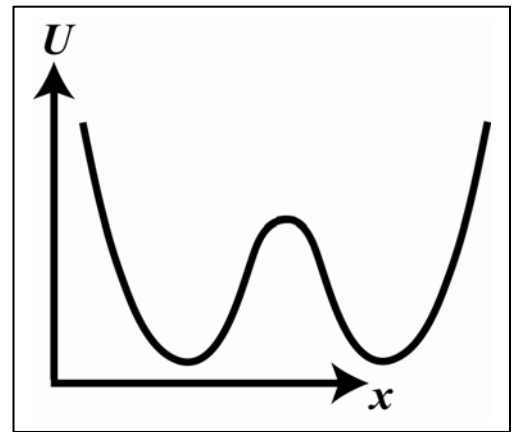
**b. (10 points)** Using the basis you defined in part a, write the matrix form of the total  $\hat{S}_z$  operator, and use it to operate on a state of your choice, showing that it gives the expected result.

**c. (10 points)** Using the notation  $|s, m\rangle$ , list the two-particle states that are simultaneous eigenstates of the total  $\hat{S}_z$  and  $\hat{S}^2$  operators, where  $s$  is the quantum number for  $\hat{S}^2$  and  $m$  is the quantum number for  $\hat{S}_z$ . (Important: I am not asking you to find these states in the basis from part a.) Also, count up the states you list in this part of the problem, and show that the number is the same as the number of basis states from part a.

**d. (10 points)** Choose one of the states from part c, and show it as clearly as you can using the style of pictures we used in class to illustrate the singlet and triplet states. Quantitatively calculate all the angles involved. State explicitly the ways in which your picture might be misleading.

**2. (20 points)** An electron is placed in a magnetic field  $\mathbf{B} = B_0 \hat{\mathbf{k}}$ . At  $t = 0$ , the electron is in the state  $|\psi(t=0)\rangle = |+\mathbf{y}\rangle$ . Use the time translation operator to find  $|\psi(t)\rangle$ . Also explain qualitatively what your result means about the time evolution of the spin state of the electron.

**3. (20 points)** An electron is confined by the symmetric double-well potential energy function shown here. The barrier between the two wells is high compared to the energy of the electron, but not infinitely high. Show that, if the electron is initially localized in the right well, it will oscillate between the two wells. (Present a complete argument, starting from basic principles.)



**4. (20 points total)**

**a. (10 points)** The Hamiltonian for a particular particle is  $\hat{H} = \omega_0 \hat{S}_x$ .

Show that  $\frac{d\langle S_y \rangle}{dt} = -\omega_0 \langle S_z \rangle$ .

**b. (10 points)** Using what you know about precession, and assuming whatever initial state for the particle you prefer, explain why this result makes qualitative sense.

**END OF EXAM**