

Exam Guidelines:

1. **IMPORTANT:** Shortly before beginning the exam, you must check your e-mail, and also send an e-mail to wsmith@haverford.edu to notify me that you're about to start. This way, if there are last minute corrections, it will be easy to distribute them to the rest of the class. If you find something on the exam which is erroneous or ambiguous, call me immediately (Office: 610-896-1332, Home 610-896-1565), even if it's very late at night! Although I try extremely hard to make everything completely clear, it is sometimes difficult to anticipate the way that others might interpret the problems. If you call me, we can straighten it out, and using the e-mail mechanism above, we can **straighten it out for the other students, also.**
2. This is a 105 minute, take-home exam. It must be completed in one continuous sitting (i.e., the clock doesn't stop if you take a break).
3. You may prepare one sheet with up to 15 equations to refer to during the test; this may not contain any words or figures. Other than this one sheet, no books, notes, etc. of any kind are permitted.
4. **You should have a calculator to take this exam.** If your calculator has graphing, programmable, or symbolic algebra/calculus features, you may not use these during the exam.
5. This exam contains 6 problems. When you're ready to begin, please check that all are present.
6. For each problem, circle your final answer.
7. The exam is to be slid under my office door (at HC or BMC) by 4 pm Sunday. Please do not turn in this exam itself or your equation sheet; only turn in your examination book. You should carefully keep this exam for future reference when your graded exam is returned.
8. Please note your starting and ending times on the front cover of the examination book.
9. Use three significant digits on all problems, unless otherwise indicated.

Some Words of Advice:

I look forward to giving you partial credit for your work. To receive credit, present your work in a clear, readable format. If you find yourself stuck on a problem, don't panic. Instead, carefully explain what you do know about the problem, what you think is going on, and how you might proceed if you could somehow get yourself "unstuck" from the part of the problem that is giving you trouble. Remember, make sure you clearly explain what you write down, since I cannot give partial credit for things I find ambiguous, or for simply copying down equations from your summary sheet. **SHOW ALL YOUR WORK! SHOW ALL YOUR WORK IN THE EXAMINATION BOOKLET!** Be sure to label your work with the problem number.

IMPORTANT: YOU WILL FIND SOME MULTIPLE-PART PROBLEMS ON THE EXAM. EVEN IF YOU CAN'T GET THE FIRST PART OF SUCH A PROBLEM, IT IS ALMOST ALWAYS POSSIBLE TO GO ON AND DO THE OTHER PARTS!!!!!!!

There will be some time pressure on this exam. Use the time available wisely.

There are 100 points total available on the exam.

After You've Completed the Exam:

1. Indicate your ending time on the front cover of your examination booklet.
2. Please sign the honor code pledge on the front cover of your examination booklet.
3. Please do not discuss ANY aspect of this exam with your classmates until I tell you it's okay (this includes, for example, even such things as whether the exam was easy or difficult, long or short, etc.).

1. (15 points) Explain fully what is meant by “orthonormal basis”. Keep your explanation as general as possible, but illustrate it with a specific example from quantum mechanics.

2. (15 points) Each of the following operators is defined for a spin 1/2 system. Each corresponds to an operator that we have encountered, but their names have been changed, and they’re expressed in bra-ket notation instead of matrix notation. For each, give the name that we have normally used for it, and briefly explain what each does. *Hint: Try applying them to eigenstates.*

$$\hat{A} = \frac{\hbar}{2} | +z \rangle \langle +z | - \frac{\hbar}{2} | -z \rangle \langle -z | \quad \hat{B} = \hbar | +z \rangle \langle -z | \quad \hat{C} = \hbar | -z \rangle \langle +z |$$

3. (15 points) The ket for a polarization state rotated by an angle ϕ relative to the x -axis is

$$|x'\rangle = \cos\phi |x\rangle + \sin\phi |y\rangle. \text{ The kets for left- and right-circularly polarized light are } |L\rangle = \frac{1}{\sqrt{2}}(|x\rangle - i|y\rangle) \text{ and}$$

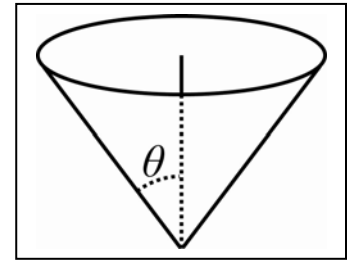
$$|R\rangle = \frac{1}{\sqrt{2}}(|x\rangle + i|y\rangle). \text{ Show that the probability that a right-circularly polarized photon will pass through a}$$

linear polarizer is independent of the angle ϕ at which the polarizer is oriented, and find this probability.

4. (15 points) We have discussed how the best picture for spin angular momentum is that it’s “delocalized around a cone”.

a. What is the cone angle (as defined in the figure) for a spin 1/2 particle in the state $| +z \rangle$?

b. What is the cone angle for a particle in the state $|1, 1\rangle$?



5. (10 Points total)

a. (5 Points) Describe an experiment that shows that the Hilbert space vectors $| +z \rangle$ and $| -z \rangle$ describing states of a spin 1/2 particle must be orthogonal

b. (5 Points) Describe an experiment that shows that the Hilbert space $| +z \rangle$ and $| +x \rangle$ describing states of a spin 1/2 particle must not be orthogonal

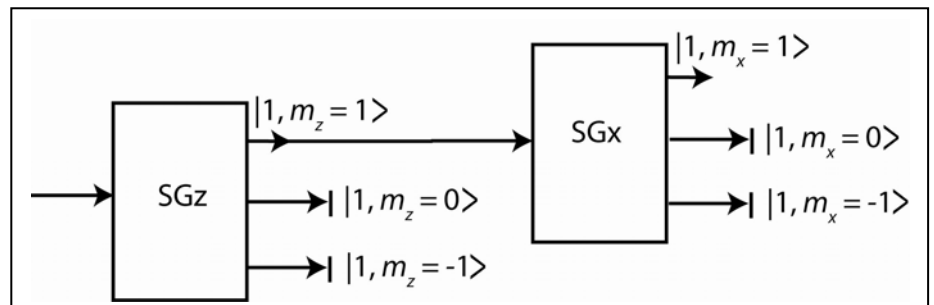
6. (30 points total) In problem (3.15), you showed that

$$|1, m_x = 1\rangle \xrightarrow{z\text{-basis}} \frac{1}{2} \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}, |1, m_x = 0\rangle \xrightarrow{z\text{-basis}} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, |1, m_x = -1\rangle \xrightarrow{z\text{-basis}} \frac{1}{2} \begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$$

a. (15 points) In the experiment shown here, what is the probability amplitude for a particle in the $|1, m_z = 1\rangle$ beam to be detected at the output in the $|1, m_x = 1\rangle$ beam?

b. (15 points) Show that

$\hat{R}(\pi \hat{\mathbf{i}}) |1, m_z = 1\rangle$ gives the expected result, to within an overall phase factor.



END OF EXAM